Third United Nations International Conference on the Peaceful Uses of Atomic Energy

A/CONF.28/A/842
FINLAND
February 1964
Original: ENGLISH

Confidential until official release during Conference

Abstract

Generalized Potential Theory for Multigroup Diffusion in a General Multiregion Reactor

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This article contains the mathematical background for a new approach to the multigroup diffusion equations in general geometries, in many respects analogous to the classical potential theory. This approach has been referred to by the author in a previous paper.

The partial differential equations describing the static or kinetic behaviour of a multiregion reactor in diffusion approximation can be transformed into a system of linear integral equations for the surface fluxes and for the normal derivatives of the fluxes at the surfaces. If the interfaces include corners or concentrated source or other singularities, the role of the normal derivative will be played by an additive set function, from which arises a Radon-measure in the corresponding integral equations. The Green functions used are those for an infinite medium of the operators

$$A_i = \nabla^2 - \varkappa_i^2$$

in the static case, or,

$$A_i = \nabla^2 - \varkappa_i^2 - \frac{1}{c_i^2}\,\frac{\partial}{\partial t}$$

in the kinetic case. The singular Green functions and their derivatives normal to the surfaces appear as kernels in the equations.

In the light of potential theory every kinetic behaviour of a reactor can be interpreted as a result of either a change in the shape of the reactor regions (moving boundaries) or a variation with time of the boundary conditions at the stationary surfaces (time-dependent extrapolation length etc.). Hence even in the kinetic case the fluxes $\Phi_i(x,y,x,t)$ can be represented by means of generalized potentials of single and double layers distributed over surfaces in the (x,y,z,t)-space.

In both main cases the resulting set of integral equations can be written as a linear equation

$$x - Kx = f \qquad x,f \in E$$

for an element x of a certain Banach space E. The vector x contains components, which are several functions defined over the surfaces present in the reactor. K is a linear bounded transformation of the space onto itself. Even for the most general geometries there are standard methods for the numerical solution of the previous equation, but in some particularly simple cases analytical solutions can be found. This formulation is especially powerful in explaining some theoretical aspects of the solutions.

A short review is first given on the various types of boundary conditions of diffusion theory. Special attention is paid to the boundary layer representations connected with these conditions. Then the whole formalism used in this paper is established. Methods of numerical solution are discussed and some possible extensions of the theory presented. Finally the connection of this method with other approaches is considered, and its advantages and disadvantages are surveyed.